Unit 2: Limits at Infinity

Name: _____

This refers to the limits that occur when our x-value approaches $+\infty$ or $-\infty$.

$$\lim_{x\to\infty} f(x) = L \quad \text{OR} \quad \lim_{x\to-\infty} f(x) = L$$

Finding Limits at Infinity

Procedure:

1. Verify that the limit is approaching ∞.
2. Divide the numerator and denominator by the highest power of x that appears in the **denominator.**
3. Algebraically simplify, until it's safe to plug in ∞ or $-\infty$.

Examples:

1. $\lim_{x\to\infty} \dfrac{6x^2+1}{2x^2-7}$

$$\lim_{x\to\infty} \frac{\frac{6x^2}{x^2}+\frac{1}{x^2}}{\frac{2x^2}{x^2}-\frac{7}{x^2}} \quad \Rightarrow \quad \lim_{x\to\infty} \frac{\frac{6x^2}{x^2}+\frac{1}{x^2}}{\frac{2x^2}{x^2}-\frac{7}{x^2}}$$

$$\lim_{x\to\infty} \frac{6+\frac{1}{x^2}}{2-\frac{7}{x^2}} \quad \Rightarrow \quad \lim_{x\to\infty} \frac{6+\frac{1}{\infty^2}}{2-\frac{7}{\infty^2}}$$

$$\lim_{x\to\infty} \frac{6+0}{2-0} = \mathbf{3}$$

"The limit of $\dfrac{6x^2+1}{2x^2-7}$ as x approaches infinity is 3."

Name: _____

2. $\lim_{x\to-\infty}\dfrac{x-5}{x^2+11}$

$$\lim_{x\to-\infty}\frac{\frac{x}{x^2}-\frac{5}{x^2}}{\frac{x^2}{x^2}+\frac{11}{x^2}} \quad\Rightarrow\quad \lim_{x\to-\infty}\frac{\frac{x}{x^2}-\frac{5}{x^2}}{\frac{x^2}{x^2}+\frac{11}{x^2}}$$

$$\lim_{x\to-\infty}\frac{\frac{1}{x}-\frac{5}{x^2}}{1+\frac{11}{x^2}} \quad\Rightarrow\quad \lim_{x\to-\infty}\frac{\frac{1}{-\infty}-\frac{1}{(-\infty)^2}}{1+\frac{11}{(-\infty)^2}}$$

$$\lim_{x\to-\infty}\frac{0-0}{1+0} = \mathbf{0}$$

---

3. $\lim_{x\to\infty}\dfrac{x^3+5}{x^2-x+1}$

$$\lim_{x\to\infty}\frac{\frac{x^3}{x^2}+\frac{5}{x^2}}{\frac{x^2}{x^2}-\frac{x}{x^2}+\frac{1}{x^2}} \quad\Rightarrow\quad \lim_{x\to\infty}\frac{\frac{x^3}{x^2}+\frac{5}{x^2}}{\frac{x^2}{x^2}-\frac{x}{x^2}+\frac{1}{x^2}}$$

$$\lim_{x\to\infty}\frac{x+\frac{5}{x^2}}{1-\frac{1}{x}+\frac{1}{x^2}} \quad\Rightarrow\quad \lim_{x\to\infty}\frac{\infty+\frac{5}{\infty^2}}{1-\frac{1}{\infty}+\frac{1}{\infty^2}}$$

$$\lim_{x\to\infty}\frac{\infty+0}{1-0+0} = \infty$$

Name: _____

4. $\lim_{x\to\infty} \dfrac{4x^2-5}{-2x+1}$

$$\lim_{x\to\infty} \frac{\frac{4x^2}{x}-\frac{5}{x}}{\frac{-2x}{x}+\frac{1}{x}} \quad \Rightarrow \quad \lim_{x\to\infty} \frac{\frac{4x^2}{x}-\frac{5}{x}}{\frac{-2x}{x}+\frac{1}{x}}$$

$$\lim_{x\to\infty} \frac{4x-\frac{5}{x}}{-2+\frac{1}{x}} \quad \Rightarrow \quad \lim_{x\to\infty} \frac{4(\infty)-\frac{5}{\infty}}{-2+\frac{1}{\infty}}$$

$$\lim_{x\to\infty} \frac{\infty-0}{-2+0} = -\infty$$

---

5. $\lim_{x\to\infty} \dfrac{x^5+3x-1}{x^3-5x+4}$

$$\lim_{x\to\infty} \frac{\frac{x^5}{x^3}+\frac{3x}{x^3}-\frac{1}{x^3}}{\frac{x^3}{x^3}-\frac{5x}{x^3}+\frac{4}{x^3}} \quad \Rightarrow \quad \lim_{x\to\infty} \frac{\frac{x^5}{x^3}+\frac{3x}{x^3}-\frac{1}{x^3}}{\frac{x^3}{x^3}-\frac{5x}{x^3}+\frac{4}{x^3}}$$

$$\lim_{x\to\infty} \frac{x^2+\frac{3}{x^2}-\frac{1}{x^3}}{1-\frac{5}{x^2}+\frac{4}{x^3}} \quad \Rightarrow \quad \lim_{x\to\infty} \frac{\infty^2+\frac{3}{\infty^2}-\frac{1}{\infty^3}}{1-\frac{5}{\infty^2}+\frac{4}{\infty^3}}$$

$$\lim_{x\to\infty} \frac{\infty^2+0-0}{1-0+0} = \infty$$

Unit 2: Limits at Infinity

Name: _____

This refers to the limits that occur when our x-value approaches $+\infty$ or $-\infty$.

$$\lim_{x\to\infty} f(x) = L \quad \text{OR} \quad \lim_{x\to-\infty} f(x) = L$$

Finding Limits at Infinity

Procedure:

1. _____
2. _____
   _____
3. _____

Examples:

1. $\lim_{x\to\infty} \dfrac{6x^2+1}{2x^2-7}$

---

2. $\lim_{x\to\infty} \dfrac{x-5}{x^2+11}$

Name: _____

3. $\lim_{x\to\infty} \dfrac{x^3+5}{x^2-x+1}$

---

4. $\lim_{x\to\infty} \dfrac{4x^2-5}{-2x+1}$

---

5. $\lim_{x\to\infty} \dfrac{x^5+3x-1}{x^3-5x+4}$